DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

371. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

In a G. P. of an odd number of terms, all of the terms being positive, and the ratio different from 1, show that the middle term is less than the arithmetical mean.

I. Solution by ELIJAH SWIFT, Princeton University.

We have a G. P. of an odd number of positive terms, a, ar, ar^2, ..., ar^{2n}.

To prove $ar^n < \dfrac{a(r^{2n+1}-1)}{(2n+1)(r-1)}$.

Assuming $r>1$ and clearing, we have to prove, a being positive,

$$(2n+1)(r^{n+1}-r^n) < r^{2n+1}-1; \text{ or}$$
$$r^{2n+1}-(2n+1)r^{n+1}+(2n+1)r^n-1>0).$$

Call the expansion on the left $f(r)$. Then $f(1)=0$.

$$f'(r)=(2n+1)r^{n-1}\{r^{n+1}-(n+1)r+n\}.$$

Call the expression in brackets $\phi(r)$. Then $\phi(1)=0$.

$$\phi'(r)=(n+1)\{r^n-1\}.$$

Then we have, since $r>1$, $n>0$, $\phi'(r)>0$.

Since $\phi(1)=0$ and $\phi'(r)>0$, $\phi(r)>0$. But $f'(r)=(2n+1)r^{n-1}.\phi(r)$. Therefore $f'(r)>0$, and finally, $f(r)>0$.

The case where $r<1$ may be treated by reversing the inequality signs.

Also solved by H. Prime, H. C. Feemster, C. E. Githens, A. M. Harding, and the Proposer.

372. Proposed by S. LEFSCHETZ, Ph. D., University of Nebraska.

Prove that $\sum\limits_{n=1}^{n=\infty} n^2x^{n-1}=\dfrac{1+x}{(1-x)^3}$, if mod.$x<1$. (*Schlömilch.*)